Non-stationary Ambient Response Data Analysis for Modal Identification Using Improved Random Decrement Technique

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Outline

• **Introduction**
  - Engineering Problems (System Analysis)
  - Ambient Vibration
  - Modal-Identification Method
  - Literature Review

• **Non-stationary Random Decrement Algorithm**
  - Theoretical Development
  - Practical Treatment
  - Extended RDD Algorithm
  - Ibrahim Time Domain Method (ITD)

• **Simulation and Discussions**

• **Conclusions**
**Introduction**

**Direct Analysis**

![Diagram](image)

**System Identification**

![Diagram](image)
It is desirable to develop techniques for modal-parameter identification only from ambient vibration data, i.e., without the need for input measurement.
In-operation testing

In-flight measurement

Most ambient excitations are random in nature

- Stationary (white) process: probability distribution is independent of time
- Non-stationary process

Ambient Vibration
• The assumption of ambient excitation is stationary white
  ➢ Natural Excitation Technique (NExT) (James et al. [1993, 1994, and 1995]) – Correlation Technique
  ➢ Random Decrement Technique (RDD) (Cole [1971], Vandiver et al. [1982], and Bedewi [1986])

• Modal identification from ambient vibration data
  ➢ Extracting Dynamic Characteristic of a building from Ambient Vibration Measurement (Ventura et al. [2003])

  ➢ Ambient vibration-based seismic evaluation of a continuous girder bridge (Ren et al. [2004])
Literature Review

- Ambient Vibration Data Analysis for Structural Identification and Global Condition Assessment (Gul and Catbas [2008])

- System Identification of Suspension Bridge from Ambient Vibration Response (Siringoringo and Fujino [2008])

- Ambient Vibration Testing and Condition Assessment of the Paderno Arch Bridge (1889) (Gentile and Saisi [2011])

- EMD-based RDD for Modal Parameter Identification of an Existing Railway Bridge (He et al. [2011])
• Many modal-identification methods are done only from impulse or free responses of a structure

• Usually, the assumption of ambient excitation is stationary white

• How to identify the major structural modes from non-stationary ambient responses?
Theoretical Development of Nonstationary Random Decrement Technique

• Assumption: non-stationary white noise in the form of a product model.

• Non-stationary random decrement signature $\delta_{x_1,x_2}(t,t+\tau)$

$$\delta_{x_1,x_2}(t,t+\tau) = \frac{E[X_1(t)X_2(t+\tau)]}{E[X_1^2(t)]} X_1(t) = \frac{R_{x_1,x_2}(t,\tau)}{E[X_1^2(t)]} X_1(t)$$

- $\delta_{x_1,x_2}(t,t+\tau)$ is in direct proportion to the $R_{x_1,x_2}(t,\tau)$ under the assumption of Gaussian processes
- For any fixed time instant $t$, $R_{x_1,x_2}(t,\tau)$ is a sum of complex exponential functions, which is of the same form as the free-vibration decay or the impulse response of the original system.

$$R_{ij}(t,\tau) = \sum_{r=1}^{n} \frac{\varphi_{ir} A_{jr}(t)}{m_r \omega_{dr}} \exp(-\xi_r \omega_{nr} \tau) \sin(\omega_{dr} \tau + \Theta_r)$$
Practical treatment of non-stationary data

• Usually very limited data samples are available in engineering practice

• To transfer the original non-stationary responses into stationary ones
  ➢ Ergodic process
Practical treatment of non-stationary data

- Assumption: \( f(t) = \Gamma(t) \cdot w(t) \)
  - Slowly – time- varying

- The responses of the system can be approximately derived as a product model with the same amplitude-modulating function as that associated with the excitation itself.
By using interval average and then applying curve-fitting, we obtain the temporal root-mean-square function, and so the envelope function.

We can then acquire the approximate stationary responses by dividing the non-stationary responses of each DOF with the same envelope function.
Extended RDD Algorithm

- Original RDD is only good for the assumption of stationary excitation.
- Practical treatment of non-stationary force-vibration data is only applicable for the non-stationary excitation in the form of a product model of white noise and a deterministic slowly-time-varying amplitude-modulating function.
- The error involved in transforming the original nonstationary responses into the approximate response of free decay would generally lead to a distortion in the modal parameters of identification.
Extended RDD Algorithm

- The standard matrix equations of motion

\[ M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = f(t) \]

- Introduce the transformation:

\[ x(t) = \Phi \cdot q(t) = \sum_{r=1}^{2m} \phi_r \cdot q_r(t) \]

- Express \( q_r(t) \) by combining the homogenous solution \( q_{rh}(t) \) and the particular solution \( q_{rp}(t) \)

\[ q_r(t) = q_{rh}(t) + q_{rp}(t) \]

\[ = e^{-\xi_r \omega_r t} \left[ q_{0r} \cos \omega_{dr} t + \frac{\dot{q}_{0r} + \xi_r \omega_r q_{0r}}{\omega_{dr}} \sin \omega_{dr} t \right] + \int_0^t \phi_r^T f(\tau) h_r(t - \tau) d\tau \] (a)

- Nonstationary zero-mean random excitation
Extended RDD Algorithm

• The modal velocity response $\dot{q}_r(t)$ results from the calculation of the first differentiation of $q_r(t)$ with respect to $t$

$$\dot{q}_r(t) = e^{-\xi_t \omega t}[\frac{\dot{q}_0 r + \xi_r \omega q_0 r}{\omega_{dr}} \sin \omega_{dr} t + (-q_0 r \omega_{dr} \sin \omega_{dr} t + \frac{\dot{q}_0 r + \xi_r \omega q_0 r}{\omega_{dr}} \omega_{dr} \cos \omega_{dr} t)] + \dot{q}_{rp}(t)$$

$$= -\xi_r \omega q_r(t) + e^{-\xi_t \omega t}(-q_0 r \omega_{dr} \sin \omega_{dr} t + \frac{\dot{q}_0 r + \xi_r \omega q_0 r}{\omega_{dr}} \omega_{dr} \cos \omega_{dr} t) + \dot{q}_{rp}(t)$$

• The following equation can then be derived

$$\dot{q}_r(t) + \xi_r \omega q_r(t) - \dot{q}_{rp}(t) = e^{-\xi_t \omega t}(-q_0 r \omega_{dr} \sin \omega_{dr} t + \frac{\dot{q}_0 r + \xi_r \omega q_0 r}{\omega_{dr}} \omega_{dr} \cos \omega_{dr} t)$$

(b)
Extended RDD Algorithm

• Substituting $t$ into $t + \tau$

$$q_r(t + \tau) = e^{-\xi_r \omega_r t} \cdot e^{-\xi_r \omega_r \tau} [\cos \omega_{dr} t (q_{0r} \cos \omega_{dr} \tau + \frac{\dot{q}_{0r} + \xi_r \omega_r q_{0r}}{\omega_{dr}} \sin \omega_{dr} \tau)$$

$$+ \frac{\sin \omega_{dr} t}{\omega_{dr}} (-q_{0r} \omega_{dr} \sin \omega_{dr} \tau + \frac{\dot{q}_{0r} + \xi_r \omega_r q_{0r}}{\omega_{dr}} \omega_{dr} \cos \omega_{dr} \tau)] + q_{rp}(t + \tau)$$

\( \text{(c)} \)

• Through insertion of Eqs.(a) and (b) into Eq.(c), the following equation can then be derived

$$q_r(t + \tau) = e^{-\xi_r \omega_r t} \left[ q_{tr} \cos \omega_{dr} t + \frac{\dot{q}_{tr} + \xi_r \omega_r q_{tr}}{\omega_{dr}} \sin \omega_{dr} t \right] - e^{-\xi_r \omega_r t} \left[ q_{trp} \cos \omega_{dr} t + \frac{\dot{q}_{trp} \sin \omega_{dr} t}{\omega_{dr}} \right] + q_{rp}(t + \tau)$$

\( \text{(d)} \)
Extended RDD Algorithm

• Random-decrement averaging

Through the ensemble averaging of $N$ pre-selected sample segments of the response measurement, the following time function can be obtained as follows

$$\delta(\tau) = \frac{1}{N} \sum_{i=1}^{N} q_r(t_i + \tau)$$

(e)
In the proposed method, since the input excitation is assumed to be a nonstationary random process with zero-mean, the behavior due to force vibration will be vanished by performing the random-decrement averaging.

\[
\delta(\tau) = \frac{1}{N} \sum_{i=1}^{N} e^{-\xi_i\omega_i \tau} \left[ q_{t,r} \cos \omega_{dr} \tau + \frac{\dot{q}_{t,r} + \xi_r \omega_r q_{t,r}}{\omega_{dr}} \sin \omega_{dr} \tau \right] + \frac{1}{N} \sum_{i=1}^{N} e^{-\xi_i\omega_i \tau} \left[ q_{t,rp} \cos \omega_{dr} \tau + \frac{\dot{q}_{t,rp} \sin \omega_{dr} \tau}{\omega_{dr}} \right] + \frac{1}{N} \sum_{i=1}^{N} q_{rp}(t_i + \tau)
\]

implies that \( \delta(\tau) \) contains the behavior of free-decay vibration

\[
\Theta(\tau) = -e^{-\xi_r\omega_r \tau} \left[ \frac{1}{N} \sum_{i=1}^{N} q_{rp}(t_i) \cos \omega_{dr} \tau + \frac{\xi \omega_n \sum_{i=1}^{N} q_{rp}(t_i) - \sum_{i=1}^{N} \dot{q}_{rp}(t_i)}{\omega_{dr}} \sin \omega_{dr} \tau \right] + \frac{1}{N} \sum_{i=1}^{N} q_{rp}(t_i + \tau)
\]

corresponds to the behavior due to force vibration
Extended RDD Algorithm

• It has been shown that an improvement to the random decrement algorithm is presented for modal identification from zero-mean nonstationary ambient vibration data.

• We can perform the random-decrement averaging over the nonstationary-responses samples to obtain good improved RDD signatures, as quasi free-vibration data, for further modal identification without any additional treatment of transforming the original nonstationary responses into stationary ones.

• Therefore, we avoid a distortion in the modal parameters of identification induced by the error involved in the approximate quasi-stationary response obtained through curve-fitting technique.
Ibrahim Time-Domain Method (ITD)

~Using free-decay responses of a structure to identify its modal parameters in complex form

- [X]: data matrix from free responses

\[ x_{ij} \equiv x_i(t_j) = \sum_{n=1}^{2m} \varphi_{ir} e^{\lambda_r t_j} \]

- [Y]: data matrix from time-delay sampling

\[ y_{ij} \equiv x_i(t_j + \Delta t) = \sum_{r=1}^{2m} \varphi_{ir} e^{\lambda_r (t_j + \Delta t)} = \sum_{r=1}^{2m} \psi_{ir} e^{\lambda_r t_j} \]

\[ \psi_{ir} = \varphi_{ir} e^{\lambda_r \Delta t} \]
Define

\[ [A][X] = [Y] \]

Least square via pseudo inverse

\[ [A] = [Y] [X]^T ([X] [X]^T)^{-1} \]

Characteristic root of the original vibration system:

\[ \lambda_r = a_r + ib_r \]

Eigenvalue of system matrix \([A]\):

\[ \rho_r = \beta_r + i\gamma_r \]

Natural freq.: \quad \text{Damping ratio:}

\[ \omega_{nr} = \sqrt{a_r^2 + b_r^2} \quad \zeta_r = \frac{|a_r|}{\sqrt{a_r^2 + b_r^2}} \]

\[ a_r = \frac{1}{2\Delta t} \ln(\beta_r^2 + \gamma_r^2) \]

\[ b_r = \frac{1}{\Delta t} \tan^{-1} \left( \frac{\gamma_r}{\beta_r} \right) \]
Non-stationary responses in the form of a product model

- Curve Fitting
- Random Decrement Algorithm
- Modal-Identification Method

Approximate stationary responses

Approximate free-decay responses

Modal parameters
Simulation and Discussions
(1/10)

• 6-DOF chain model
  ➢ The non-proportionally damped system
  ➢ Non-stationary excitation acts on 6th mass point of the system
  ➢ Non-stationary excitation employs product model

\[
 f(t) = \mathcal{E}(t) \cdot w(t)
\]
6-DOF chain model

\[
[M] = \begin{bmatrix}
2 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 4 \\
\end{bmatrix}
\]

\[
[K] = 600 \cdot \begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & -1 & 3 & -2 \\
0 & 0 & 0 & 0 & 0 & -2 \\
\end{bmatrix}
\]

\[
[C] = 0.05[M] + 0.001[K] + 0.2 \begin{bmatrix}
1 & . & . \\
. & . & . \\
1 & . & 1 \\
\end{bmatrix}_{6 \times 6}
\]
Simulation and Discussions
(3/10)

- Displacement response in 1\textsuperscript{st}, 3\textsuperscript{rd}, 5\textsuperscript{th} D.O.F.

- Fourier spectrum of displacement response in 1\textsuperscript{st}, 3\textsuperscript{rd}, 5\textsuperscript{th} D.O.F.
Fig. 5. Typical displacement responses of the 6-DOF chain system subject to nonstationary white input and the corresponding random dec signatures of each DOF through the conventional RDD (in conjunction with a curve-fitting technique) as well as the improved RDD.
Simulation and Discussions (5/10)

- Results of modal-parameter identification
  - ITD method in conjunction with the conventional RDD and curve-fitting technique

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<th>Mode</th>
<th>Natural Frequency (rad/s)</th>
<th>Damping Ratio (%)</th>
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<td>6</td>
<td>33.73</td>
<td>33.21</td>
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- ITD method in conjunction with the improved RDD

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<td>31.39</td>
</tr>
<tr>
<td>6</td>
<td>33.73</td>
<td>33.35</td>
</tr>
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</table>
Simulation and Discussions (6/10)

- 8-DOF truss model

\[
M = \begin{bmatrix}
100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \\
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
27071.1 & 0 & 0 & 0 & 0 & -10000 & 0 & -3535.5 & -3535.5 \\
0 & 17071.1 & 0 & -10000 & 0 & 0 & -3535.5 & -3535.5 \\
0 & 0 & 27071.1 & 0 & -3535.5 & 3535.5 & -10000 & 0 \\
0 & -10000 & 0 & 17071.1 & 3535.5 & -3535.5 & 0 & 0 \\
-10000 & 0 & -3535.5 & 3535.5 & 27071.1 & 0 & 0 & 0 \\
0 & 0 & 3535.5 & -3535.5 & 0 & 17071.1 & 0 & -10000 \\
-3535.5 & -3535.5 & -10000 & 0 & 0 & 0 & 27071.1 & 0 \\
-3535.5 & -3535.5 & 0 & 0 & 0 & -10000 & 0 & 17071.1 \\
\end{bmatrix}
\]

\[
C = 0.17M + 0.001K \, \text{N} \cdot \text{sec/m}.
\]
Simulation and Discussions (7/10)

• Results of modal-parameter identification

  ITD method in conjunction with the conventional RDD and curve-fitting technique

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<td>18.42</td>
<td>18.34</td>
<td>0.44</td>
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<tr>
<td>8</td>
<td>20.43</td>
<td>20.33</td>
<td>0.50</td>
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ITD method in conjunction with the improved RDD

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<td>20.43</td>
<td>20.36</td>
<td>0.35</td>
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Simulation and Discussions (8/10)

- 7-DOF model (containing two pairs of close modes)

\[
M = \begin{bmatrix}
2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3.5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\text{N} \cdot \text{s}^2 / \text{m},
\]

\[
K = 400 \cdot \begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2.7 & -1 & -0.7 & 0 & 0 & 0 \\
0 & -1 & 3 & -1 & 0 & 0 & 0 \\
0 & -0.7 & -1 & 3.7 & -1 & -1 & 0 \\
0 & 0 & 0 & -1 & 2 & 0 & -1 \\
0 & 0 & 0 & -1 & 0 & 2.85 & -1 \\
0 & 0 & 0 & 0 & -1 & -1 & 2 \\
\end{bmatrix}
\text{N} / \text{m},
\]

\[
C = 0.2M + 0.001K \quad \text{N} \cdot \text{s} / \text{m}
\]

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Simulation and Discussions (9/10)

- For higher DOF structural system excited by a sample of the seismic record

Assume that each mass is 1 kg and all spring constants are 600 N/m. The damping matrix of the system is assumed to be

\[
C = 0.05M + 0.001K + 0.02\begin{bmatrix}
1 & \ldots & 1 \\
\vdots & \ddots & \vdots \\
1 & \ldots & 1 \\
\end{bmatrix}_{20 \times 20} \text{ N} \cdot \text{s/m}
\]

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</tr>
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<td>46.74</td>
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<td>48.30</td>
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</tbody>
</table>
## Simulation and Discussions

<table>
<thead>
<tr>
<th>Mode (E6XR3X) (0~100Hz)</th>
<th>Natural Frequency (Hz)</th>
<th>Damping Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shock Vibration</td>
<td>Ambient Vibration</td>
</tr>
<tr>
<td>1</td>
<td>26.37</td>
<td>25.17</td>
</tr>
<tr>
<td>2</td>
<td>38.09</td>
<td>36.89</td>
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<tr>
<td>3</td>
<td>49.07</td>
<td>48.34</td>
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<tr>
<td>4</td>
<td>82.76</td>
<td>80.76</td>
</tr>
</tbody>
</table>
Conclusions

- Innovative methods have been proposed in this research for determination of the modal parameters of a structure from its measured ambient response data only.

- It is shown theoretically that the nonstationary response signals can be converted into free-vibration data via the random decrement technique by assuming the ambient excitation to be nonstationary white noise (product model).
A technique of curve-fitting is proposed in conjunction with the random decrement technique for modal identification. It can be done, in practice, under the assumption of slowly-time-varying amplitude-modulating envelope function.

However, the error involved in the approximate free-decay response would generally lead to a distortion in the modal identification.

If the ambient excitation can be modeled as a zero-mean nonstationary process, without any additional treatment of transforming the original nonstationary responses, the nonstationary cross random dec signatures of structural response are shown in the same mathematical form as that of free vibration of a structure, from which modal parameters of the original system can thus be identified.
• The choice of the reference channel is significant to compute the random decrement signatures. The reference channel is chosen as a response channel whose Fourier spectrum has rich frequency content around the structure modes of interest. The richer frequency content the reference channel has, the better results of modal parameters identification can be achieved.

• Numerical simulations, including one example of using the practical excitation data, confirm the validity of the proposed method for identification of modal parameters from nonstationary ambient response data.
Thanks for your kind attention!

The End